Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at http://about.jstor.org/participate-jstor/individuals/early-journal-content.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

subject either in the pure or the applied field. It is conceivable that these courses are not the best for high-school teachers of mathematics, especially for those who do not pursue their studies in course beyond the college. It is quite likely that there should be more survey courses even at the expense of intensive work over a narrow field. It is quite certain that the emphasis should be changed, giving more geometrical, historical and pedagogical courses. Hence I offer the above complement to Question No. 27."

Replies to Question No. 27 were published in the December and January issues of the MONTHLY.

31. In the light of questions 27 and 30, the Editors wish to propose that a symposium be called for on the question: What are the actual courses now offered in colleges and universities in this country for the preparation of teachers of mathematics (1) for secondary schools, (2) for colleges? The discussion may well lead to the consideration also of what courses *should be offered* for the preparation of teachers of mathematics (1) for secondary schools, and (2) for colleges.

DISCUSSIONS.

I. RELATING TO A SIMPLE PROOF BY INDUCTION OF AN INTERESTING NUMBER RELATION.

BY CHARLES R. DINES, Dartmouth College.

THEOREM: *For any set* $(a_i \mid i = 1, 2, \cdots, n)$ *of* n *distinct numbers, real or imaginary* $(n > 1)$, *we have*

$$\sum_{i=1}^{n}\prod_{\substack{j=1\\ j\neq i}}^{n}\frac{1}{a_i - a_j} = 0. \tag{I}$$

Proof: (A) Relation (I) holds for the smallest admissible value of n, viz., $n = 2$.

(B) Assume that relation (I) holds when $n = r$; that is, for any set of r distinct integers,

$$\sum_{i=1}^{r}\prod_{\substack{j=1\\ j\neq i}}^{r}\frac{1}{a_i - a_j} = 0. \tag{1}$$

Let

$$\sum_{i=1}^{r+1}\prod_{\substack{j=1\\ j\neq i}}^{r+1}\frac{1}{a_i - a_j} = N. \tag{2}$$

We wish to prove that N is zero. Multiply both sides of (1) by $1/(a_{r+1} - a_1)$ and add to (2). Then

$$N = \sum_{i=1}^{r+1}\prod_{\substack{j=1\\ j\neq i}}^{r+1}\frac{1}{a_i - a_j} + \frac{1}{a_{r+1} - a_1}\sum_{i=1}^{r}\prod_{\substack{j=1\\ j\neq i}}^{r}\frac{1}{a_i - a_j}. \tag{3}$$

But

$$\prod_{j=2}^{r+1}\frac{1}{a_1 - a_j} + \frac{1}{a_{r+1} - a_1}\prod_{j=2}^{r}\frac{1}{a_1 - a_j} = 0$$

and when $1 < i < r + 1$

$$\prod_{\substack{j=1\\ j\neq i}}^{r+1}\frac{1}{a_i - a_j} + \frac{1}{a_{r+1} - a_1}\prod_{\substack{j=1\\ j\neq i}}^{r}\frac{1}{a_i - a_j} = \frac{a_{r+1} - a_1 + a_i - a_{r+1}}{(a_{r+1} - a_1)(a_i - a_{r+1})}\prod_{\substack{j=1\\ j\neq i}}^{r}\frac{1}{a_i - a_j},$$

which finally reduces to

$$\frac{1}{a_{r+1}-a_1}\prod_{\substack{j=2\\ j\neq i}}^{r+1}\frac{1}{a_i-a_j}.$$

Thus (3) may be written

$$\begin{aligned} N &= \prod_{j=1}^{r}\frac{1}{a_{r+1}-a_j}+\frac{1}{a_{r+1}-a_1}\sum_{i=2}^{r}\prod_{\substack{j=2\\ j\neq i}}^{r+1}\frac{1}{a_i-a_j} \\ &= \frac{1}{a_{r+1}-a_1}\left[\prod_{j=2}^{r}\frac{1}{a_{r+1}-a_j}+\sum_{i=2}^{r}\prod_{\substack{j=2\\ j\neq i}}^{r+1}\frac{1}{a_i-a_j}\right] \\ &= \frac{1}{a_{r+1}-a_1}\sum_{i=2}^{r+1}\prod_{\substack{j=2\\ j\neq i}}^{r+1}\frac{1}{a_i-a_j}. \end{aligned}$$

Then using relation (1) in form applicable to the set $(a_i \mid i = 2, 3, \cdots, r+1)$ we see that N is zero and, by induction, relation (I) holds as stated.

This proof of a simple relation between numbers is of interest in an elementary way in algebra. While the theorem may be of no particular value in applications, its proof by induction brings in certain elements which are not often found in a proof by that method. The ordinary assumption is used twice in the proof, whereas, in most, if not all, other induction proofs it is used only once.

A simple proof of the theorem may be given if we presuppose the complex variable theory. For, if we have a set of distinct numbers $(a_1, a_2, \cdots, a_n)$, $n > 1$, the quantity

$$\prod_{\substack{j=1\\ j\neq i}}^{n}\frac{1}{a_i-a_j}$$

is, for every i, just the residue at the point a_i of

$$\prod_{j=1}^{n}\frac{1}{z-a_j}$$

and the sum of these residues is zero, so that the theorem is immediate.

II. RELATING TO THE FOLIUM OF DESCARTES.

By Maurice C. Baudin, Student at The University of Chicago.

The current methods of tracing the cubic

$$x^3 + y^3 - \mu xy = 0$$

are very complicated; so complicated indeed that their authors have found it too laborious to apply them thoroughly and draw a correct figure.[1] Dr. G. Teixeira in his *Obras sobre Mathematica* proposes a process by which, with a sensible amount

---

[1] In Dowling and Turneaure's *Analytic Geometry* the curve seems to have besides the double point at the origin, a cusp at P.